Indian Statistical Institute, Bangalore

B. Math (Hons.) Third Year

Second Semester - Differential Equations

Final Exam   Duration : 3 hours   Max Marks 50   Date : April 28, 2017

Section I: Answer any four and each question carries 6 marks.

1. Solve $(xy-1)dx + x(x-y)dy = 0$ by finding an integrating factor.
2. Suppose y_1 and y_2 are twice continuously differentiable functions on $\mathbb{R}$ such that $y_1(0)y_2'(0) \neq y_2(0)y_1'(0)$. Is there an interval I containing 0 so that y_1 and y_2 are solutions of a second order homogeneous linear differential equation on I. Justify your answer.
3. Solve $xy'' - (2x+1)y' + (x+1)y = 0$.
4. Solve $(1-x^2)y'' - xy' + p^2y = 0$ by power series method.
5. Find solutions u of the 2-dimensional heat equation that satisfy the homogeneous Dirichlet condition and are of the form $u(x,y,t) = F(x)G(y)H(t)$.
6. State and prove mean value property for harmonic functions on $\mathbb{R}$.

Section II: Answer any two and each question carries 13 marks.

1. (a) Describe the method of variation of parameters and use it to solve the equation $y'' - 2y' - 3y = 64xe^{-x}$ (*Marks: 6*).

   (b) Solve $x(1-x)y'' + (\frac{3}{2} - 2x)y' + 2y = 0$ near $x = 0$.
2. (a) State and prove the maximum principle for heat equation.

   (b) Solve $(3y-2u)u_x + (u-3x)u_y = 2x - y$, $u(s,s) = 0$ (*Marks: 6*).
3. (a) Prove the orthogonality relation between Legendre polynomials (*Marks: 6*).

   (b) Prove $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ (*Marks: 4*).

   (c) Prove or disprove that the positive zeroes of J_p and J_{p-1} alternate.